

Errol Rollox

Student #UM75894SME85076

Fluid Mechanics

Atlantic International University

02/2024

Contents	Pages
1 Introduction	1
2 Fluid Properties	1
3 Fluid Types	3
4 Fluid Flows	4
5 Reynolds Number	4
6 Incompressible Flow	5
7 Compressibility and Compressible Flow	6
8 Conclusion	8
9 Illustrations	10
10 Bibliography	11

Significant Fluid Properties and Characteristics of Fluid Flow

Introduction

Fluid Mechanics is that branch of engineering that deals with the behavior of fluids in motion or at rest. Fluid in motion is termed fluid dynamics while fluids at rest is called fluid statics.

Whether at rest or in motion fluids take the shape of the inner surface boundaries containing them. This is obvious with liquids, but with gases which are also fluids, they occupy the entire volume of their container. Since the container could be of any shape, liquid fluids will deform to conform to boundaries. This deformation is due to shear stress acting on the fluid at their boundaries. Such a stress is defined as the rate of change of velocity of a moving fluid as its depth in the container changes. That is, for example, for water flowing in a twelve-foot depth pond or waterway, the shear stress at the bottom where there is theoretically no flow, velocity of flow equals zero, therefore no stress. While at an eight-foot depth there is flow taking place (fluid velocity not equal zero), therefore shear stress is present there. Mathematically this is written:

$$\tau = \mu \cdot dv/dy, \text{ where}$$

dv represents an element of velocity

dy represents an element of depth, and

μ is a proportionality constant related to fluid viscosity.

Fluid Properties

Viscosity

This is defined as a measure of a fluid's resistance to shear or angular deformation.¹ It's a property that resists the movement of adjacent layers of the fluid which leads to the angular deformation. It relates the local stresses in a moving fluid to the strain rate of the fluid element.²

Because of movement of the layers of a fluid, the resulting viscosity is called dynamic viscosity, and symbolized, μ , which relates the shear stress to velocity gradient.³ Also note that viscosity of a liquid decreases with increasing temperature because of decreasing inter-molecular forces (cohesive forces) with increasing temperature⁴. Whenever there is an increasing temperature the density of a fluid is affected, and so also is the value of μ .

Density

This is defined as the amount of mass of fluid flowing per unit volume. With changing values of μ and a corresponding density ρ (rho), another relationship ν (nu), kinematic viscosity is found.

This is, $\nu = \mu/\rho$. The kinematic viscosity property plays a vital role in the determination of some fluid flow characteristics. Measurements of this viscosity using the SI system (metric system) shows kinematic viscosity has units L^2/T where L is the unit for length and T the unit for temperature⁵.

The units for density are kg/m^3 (mass per unit volume). Note that as the temperature of a fluid increases, its density decreases, and as the pressure increases its density increases.

Density when applied to gravity produces a particular form of weight – specific weight. This is defined as the weight of a given volume of a fluid, and is denoted by $\gamma = \rho g$. Its units are $kg/m^3 \cdot m^2/s^2$ or $kg\ m^{-2}s^{-2}$.

Specific Gravity

This is a ratio of the density of a specific fluid to the density of a standard reference fluid such as water or air; in the case of gases, it is denoted:

$$SG_{\text{liquid}} = \rho_{\text{liquid}}/\rho_{\text{water}} \quad \text{or}$$

$$SG_{\text{gas}} = \rho_{\text{gas}}/\rho_{\text{air}}$$

The liquids have close-packed molecules with strong cohesive forces, definite volume and free surfaces. The gases have widely-spaced molecules, negligible cohesive forces, and they expand to fill the container.

These properties apply to all types of fluids which in turn can be classified into four main categories, namely,

- a, Ideal fluid
- b, real fluid
- c, Newtonian fluid
- d, Non-Newtonian fluid

See Fig 1.

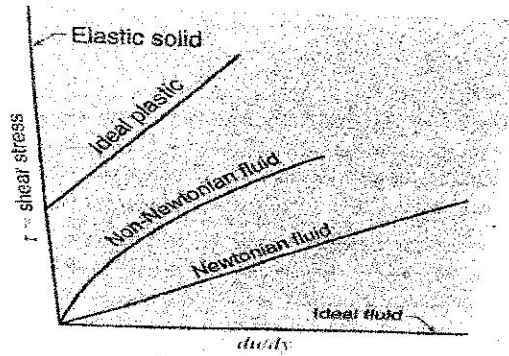


Fig 1

Fluid Types

Ideal Fluid

This is one in which there is no friction between adjacent moving layers, that is, one in which the viscosity is zero. It is also termed inviscid fluid.

Real Fluid

Here, there is the development of shear stresses between neighboring particles when they are moving at different velocities.

Newtonian Fluid

This is a fluid in which the viscous stresses are proportional to the element strain rates and the coefficient of viscosity. This is simply the rate of change of deformation over time.⁶

Non-Newtonian Fluid

These are fluids that do not obey the relationship between stress and shear strain.

Of these categories the one most used in practice is the Newtonian. This type is applied in the two main areas of fluid flow – Laminar and Turbulent. Laminar flow is seen as steady, uniform flow in which there is no macroscopic mixing between adjacent layers of fluid.⁷ On the other hand turbulent flow does lead to macroscopic mixing. See Figs. 2 and 3.

Fluid Flows

Laminar Flow

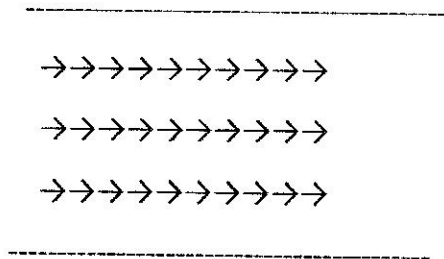


Fig. 2

Steady flow here means the velocity at any point in the fluid is constant and flow is in layers.

Turbulent Flow

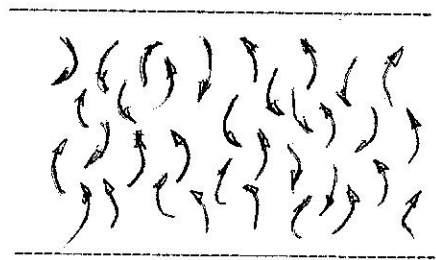


Fig. 3

Unsteady flow here shows fluid particles curved away from streamlines and mixing with adjacent layers.

The common characteristic of both flows is layer distortion. They conform quite easily to the shape of their container, whether the flow is laminar or turbulent. The determination of turbulence or laminarity is often done by application of a mathematical formula for a well-known flow parameter called Reynolds number. It is a dimensionless number comparing inertial forces to viscous forces in a flowing fluid.

Reynolds Number

The formula generally used for this number is:

$$Re_{\#} = \rho \cdot U \cdot L / \mu$$

where ρ = density,

U is the velocity of the fluid

L the length of flow investigated.

μ is the dynamic viscosity of the fluid

The calculated value of the Reynolds number is used to categorize the nature of the flow, be it laminar, turbulent or transitional.

If $Re < 2000$ flow is laminar

$2000 < Re < 4000$ flow is transitional

$4000 < Re$ flow is turbulent

These numbers are not exact but good approximations, depending on the specific applications.

In particular, modeling data from experimental investigations is one of the prime uses of

Reynolds number. When the number derived from an experiment's data closely matches the theoretically calculated value, then the criterium of similarity is achieved and the nature of the flow can be predicted.

Low Reynolds numbers indicate steady or uniform flow; this is more easily predictable for incompressible fluids than it is for say air. As a result, characteristics of steady flow are easily measurable and so low flow speeds and constant density values can be determined quickly.

When measurements show increasing density and/or velocity values over some length of region, it indicates a transition of flow is taking place; the flow is neither laminar nor turbulent, and so this region is called the transition region. During its length the calculated Reynolds number is greater than 2000 but less than 4000 ($2000 < Re_{\#} < 4000$). As the flow speed keeps increasing, the resulting Reynolds number also increases, moving past 4000, thereafter the flow is no longer streamlined but becomes turbulent. See Fig. 3

But if the extended region reaches some finite length "l" and flow is laminar then some work could be done at position "l" because of accumulated energy there. In particular when the flow is internal as in some pipeline systems and the fluid is an incompressible liquid, it can be treated as an ideal flow and so the known Bernoulli's theorem can be used.

Incompressible Flow

Bernoulli Theorem

This states that for a steady state ideal flow of an incompressible fluid, the total energy comprising mechanical energy associated with fluid pressure, gravitational potential energy of elevation and kinetic energy of fluid motion remains constant.⁸ That is, from point 1 to another point 2 in an ideal flow situation the total energy of flow is represented as:

$$P_1/\rho + 0.5(V_1)^2 + g_1Z_1 = P_2/\rho + 0.5(V_2)^2 + gZ_2 = \text{Constant, where } \rho \text{ is the density}$$

When pressure changes are very small in incompressible fluids and friction is also small as in some pipe flow problems, then the Bernoulli Theorem can be used quite satisfactorily for real fluids applications. However, if the flow is through very long pipelines, then this equation must be modified to account for large energy losses, called head losses. They could be friction losses h_f , pump losses h_p , or turbine losses h_t .

Now working in terms of energy per unit weight, where a specific-weight $\gamma = \rho g$, the Bernoulli's Theorem becomes:

$$P_1/\gamma + 0.5(V_1)^2/g + Z_1 = P_2/\gamma + 0.5(V_2)^2/g + Z_2 = \text{Constant}$$

When relevant losses are included during practical use we get:

$$(P/\gamma + 0.5V^2/g + Z)_{in} = (P/\gamma + 0.5V^2 + Z)_{out} + h_f + h_t - h_p$$

This approach allows the determination of volume flow rate Q, and/or pressure P.

The Q can be calculated through use of the flow velocity and the area perpendicular to the flow, applied mostly when the flow medium is liquid or incompressible.

Velocity can also be determined from Bernoulli's theorem,

when the inner diameter of the system pipe is known, combining the two produces Q.

Continuity Equation

This states $Q = AV$, and this is known as the continuity equation.

$$V = Q/A \quad \text{where } A = \pi D^2/4$$

$$\text{and } V = 4Q/\pi D^2 \quad \text{where } D \text{ is the diameter of the pipe.}$$

When the flow medium is air and its velocity is high, the density varies along the way making

The mass flow rate, $m' = \rho AV$ where m' is titled "m-dot"

and m' means the derivative with respect to time.

for continuity, $m' = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$

It is the presence of density changes that make the flow compressible.

Compressibility and Compressible Flow

Compressibility is a measure of the ease or difficulty with which a fluid's volume is changed or reduced when subjected to an external pressure. This causes the density to change. It takes place mostly when the fluid is gaseous and moving at an extremely high speed. If the fluid is liquid such compressibility effects dictate the speed at which flow can occur. If there is a small reduction in the flow pipe's diameter this can lead to the condition known as flow choking. This is a condition in which no additional mass flow can take place. It happens when the speed of the moving gas approaches Mach one, that is, when it approaches the speed of sound, which is approximately 300m/s; at this time the flow becomes compressible. At such speeds its density and pressure change significantly. Comparing the speed of the moving fluid to the speed of

sound, that ratio is called Mach Number, and written as,

$$Ma = v/c$$

where v is the speed of the fluid and " c " is the speed of sound.

Whenever $Ma < 0.3$ flow is considered incompressible and so density effects are negligible.

When $0.3 < Ma < 0.8$, flow is subsonic and density effects must be considered

When $1.2 < Ma < 3.0$, flow is supersonic and density values must be included in calculations.

And for $Ma > 3.0$, flow is hypersonic.

The flows can be external as is the case around airplanes, rockets and re-entry vehicles, or internal as happens inside ducts and nozzle passages.

For large density changes it follows from the equation of state that the pressure and temperature changes are substantial, enabling two new properties of the fluid to become active, introducing two new equations into the system and these must be solved simultaneously with equations of momentum and continuity.

Thus, four fundamental equations must be used when analyzing compressible flow.

Continuity is expressed as a mass flow rate, $m' = \rho AV = \text{constant}$

Momentum, for one-dimension steady flow is: $F = m'(V_2 - V_1) = \rho_2 Q_2 V_2 - \rho_1 Q_1 V_1$

Energy Equation for compressible flow: $(P_1/\gamma_1 + I_1 + z_1 + V_1^2/2g) + Q_H = (P_2/\gamma_2 + I_2 + z_2 + V_2^2/2g)$

where Q_H = heat lost and I = heat energy.

Equation of State: $P/\rho = p\upsilon = RT$ where $\upsilon = 1/\rho$ is a specific volume and P is the pressure.

For internal flows, every elemental change in pressure brings about a change in velocity because of the value of density at that moment.

Pressure changes are the main phenomena responsible for aerodynamic forces in external compressible flows. As is known, when the working fluid is air in motion it creates aerodynamic forces that act on an aircraft for example. See Fig. 4

These forces are identified as:

Thrust: The force acting forward that pushes the airplane ahead. In the case of today's aircraft, the thrust is generated inward of the airplane by its engines and sometimes by a very small force due to a tailwind.

Drag: This is the force that tends to hold back the forward movement of the airplane, reducing its velocity.

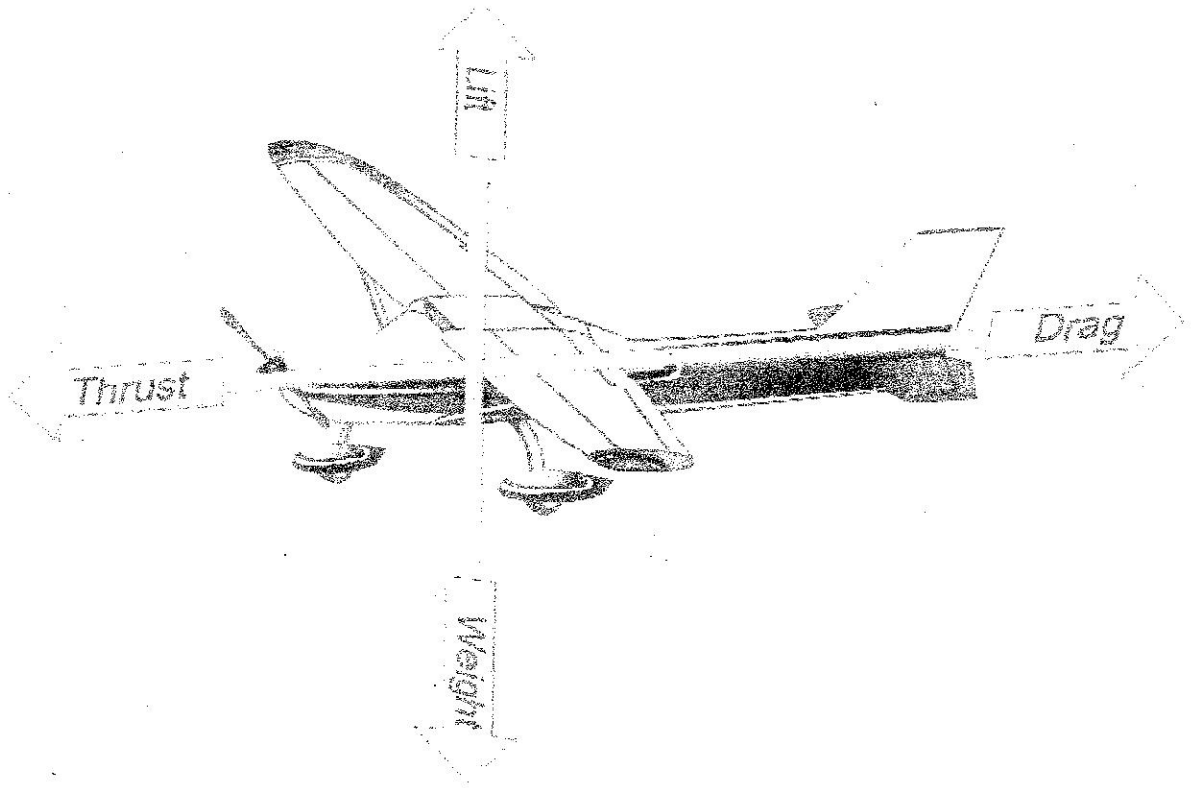


Fig. 4

Lift: This force lifts the airplane. It is achieved by the action of thrust and angle of attack of the wings.

Weight: This is a component of force acting opposite the lift component; its formula is $F = mg$ where m is the total mass of the airplane and g is the acceleration due to gravity.

These are the fluid forces acting simultaneously on an airplane in flight. In steady-level flight, the overall lift equals the weight of the aircraft while its drag is balanced by engine thrust.⁹

Therefore, it can be seen that thrust is the force that overcomes all viscous, external and pressure forces needed to propel the aircraft forward. That is,

$$F_{\text{internal}} + F_{\text{external}} = F_{\text{pressure}} + F_{\text{viscous}}$$

the total driving force is the sum of pressure and viscous forces. The pressure itself is usually easily determined by manipulation of Bernoulli's Equation, while the viscous forces can be deduced from the Navier-Stokes Equations. Navier-Stokes equations are used mainly when the flow is unsteady and compressible. For the compressible flow the momentum component of the Navier-Stokes equation is the one usually applied. It is written as:

$\rho Du_j/Dt = -\partial P/\partial x_j + \rho g_j + \mu \partial^2 u_j/\partial x^2 + (\mu_v + 1/3\mu) \partial/\partial x_j \cdot \partial u_m/\partial x_m$ ¹⁰. The presence of the density term in the equation tells that some viscous effects are present. Since the force being sought is a pressure force, then the viscous terms can be omitted from this equation reducing it to,

$$\rho Du/Dt = -\nabla_p + \rho g$$

This holds true in one dimension. But in three dimensions when flows become unsteady very quickly, applicable Navier-Stokes equations combine the changing characteristics of the flow (i.e. density change, velocity, energy and weight) with time. This is done using configurations of continuity, momentum, energy and weight. See Fig. 5.

The practical result of all this is the solution to certain flow problems, by solving all the equations simultaneously. As can be seen this entails a huge tedious task which is usually done efficiently by a program in numerical methods in the subject of Computational Fluid Dynamics (CFD). It is a very efficient and accurate way of finding reliable results.

Conclusion

It can be seen that fluid properties are directly affected by the nature of the flow.

The rate of flow invariably determines the category of application. That is, for low-speed uniform flow liquids are the better choice in many areas while gaseous fluids allow high speed applications.

When flow is at a very high speed the magnitudes of its characteristics – velocity, pressure,



Navier-Stokes Equations

3 - dimensional - unsteady

Glenn
Research
Center

Coordinates: (x,y,z) Time: t Pressure: p Heat Flux: q
 Density: ρ Stress: τ
 Velocity Components: (u,v,w) Total Energy: Et
 Reynolds Number: Re
 Prandtl Number: Pr

Continuity:
$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X - Momentum:
$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r} \left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right]$$

Y - Momentum:
$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r} \left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \right]$$

Z - Momentum:
$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r} \left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \right]$$

Energy:
$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z} - \frac{1}{Re_r Pr_r} \left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right]$$

$$+ \frac{1}{Re_r} \left[\frac{\partial}{\partial x} (u \tau_{xx} + v \tau_{xy} + w \tau_{xz}) + \frac{\partial}{\partial y} (u \tau_{xy} + v \tau_{yy} + w \tau_{yz}) + \frac{\partial}{\partial z} (u \tau_{xz} + v \tau_{yz} + w \tau_{zz}) \right]$$

On this slide we show the three-dimensional unsteady form of the Navier-Stokes Equations. These equations describe how the velocity, pressure, temperature, and density of a moving fluid are related. The equations were derived independently by G.G. Stokes, in England, and M. Navier, in France, in the early 1800's. The equations are extensions of the Euler Equations and include the effects of viscosity on the flow. These equations are very complex, yet undergraduate engineering students are taught how to derive them in a process very similar to the derivation that we present on the [conservation of momentum](#) web page.

Fig. 5

energy, etc., are tedious and difficult to calculate and so use is made of the well-known Navier-Stokes equations to determine their values. This determination is done through the use of algorithms of Computational Fluid Dynamics (CFD). However, it has been found that the solutions to these theoretical equations are still not fully understood,¹¹ especially so, because of the phenomenon of turbulence which indirectly is incorporated in the equations.

Illustrations

Fig. 1 Fluid Mechanics with Engineering Applications 10th Edition by E. John Finnemore and Joseph B. Franzini, Pub. McGraw Hill, 2002, P. 31

Fig. 4 [www.grc.nasa.gov>www>BGP/cruise.html](http://www.grc.nasa.gov/www/BGP/cruise.html)

Fig. 5 [www.grc.nasa.gov>www>BGP/navier-stokes-equation.html](http://www.grc.nasa.gov/www/BGP/navier-stokes-equation.html)

Bibliography

- 1 Finnemore, E. John and Franzini, Joseph B. Fluid Mechanics with Engineering Applications, tenth edition, Pub., McGraw Hill, 2002, P.29.
- 2 Fluid Mechanics by Frank M. White, 4th Edition, McGraw Hill Pub, 1999, P22.
- 3 Fluid Dynamics, Schaum's Outline Series by William F. Hughes and John A. Brighton, Publisher, McGraw Hill, 1967, P. 4
- 4 Fluid Mechanics by Kundu, P.K., Ira M. Cohen and David R. Dowling, with contributions from Gretar Tryggvason, 6th edition, Publisher Academic Press, 2016, P.10
- 5 Ibid., White, P. 24
- 6 Ibid., White, P. 227
- 7 J. Thewlis, Concise Dictionary of Physics and related subjects 2nd ed., Pub. Pergamon Press, 1979, P139
- 8 Bernoulli's Theorem, Encyclopedia Britannica, Publisher: Encyclopedia Britannica Inc., January 05, 2024.
- 9 Ibid Kundu, P. 779
- 10 Ibid Kundu, P. 131
- 10 <https://www.grc.gov/www/k-12/BGP/Navier-Stokesequations.html>